MAC-CPTM Situations Project

Harrington Situation 2: Arithmetic Sequences/Proof

Prompt

While exploring arithmetic sequences in a middle school Advanced Algebra class, the teacher asks, "If $A_{50} = 148$, is $A_{100} = 296$?" When the students said "no" he followed with, "Would this ever be true?" The students explored patterns and suggested that doubling the term number would double the term in the sequence 6, 12, 18, 24,… and in the sequence 4, 8, 12, 16,… and decided that if the first term was the common difference, it would work.

Mathematical Foci

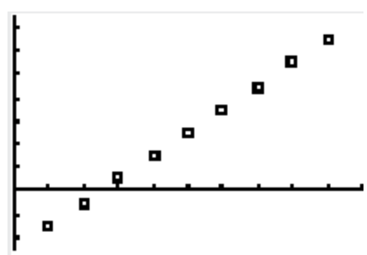
Mathematical Focus 1

The terms of an arithmetic sequence are in a linear pattern for their constant difference.

Arithmetic sequences can be defined recursively as if $A_1 = a_1$, $A_{n+1} = A_n + d$.

So if $A_1 = -3$ and $d = 2$, then the sequence is fully defined and is -3, -1, 1, 3, 5,…

With the horizontal axis representing the term number n, and the vertical axis A_n, graphically this is



Mathematical Focus 2

The nth term of an arithmetic sequence is given by $A_n = a_1 + (n - 1)d$, where a_1 is the first term in the sequence and d is the common difference.

If an arithmetic sequences can be defined recursively as if $A_1 = a_1$, $A_{n+1} = A_n + d$, and $A_1 = a_1$, then

$$
\begin{aligned}
A_2 &= a_1 + d \\
A_3 &= a_1 + d + d \\
A_4 &= a_1 + d + d + d \\
&\vdots \\
A_n &= a_1 + (n - 1)d
\end{aligned}
$$

Mathematical Focus 3

Proof is not accomplished by example and middle school Algebra students can begin to develop an understanding of how they might develop a mathematical argument to show that something is always true.

Students explored patterns and suggested that doubling the term number would double the term in the sequence 6, 12, 18, 24,… and in the sequence 4, 8, 12, 16,… and decided that if the first term was the common difference, it would work.

The students conjecture is certainly true since if $a_1 = d$ and $A_n = a_1 + (n - 1)d$

then $A_n = d + (n - 1)d$

$A_n = nd$

so $A_{2n} = 2nd = 2A_n$